where the characteristics of the same family intersect is interpreted as the beginning of the embedded shock wave structure, d) the embedded intersecting shock wave shape in the downstream direction is approximated by the envelope formed from the succeeding intersecting characteristics—extended to the jet centerline. The details of the calculation may be found in Refs. 4 and 5. Assumption d actually causes some overestimation of the freejet height due to the coarse mesh used. This overestimation may also be partially accounted for by the turbulent mixing which actually occurred at the jet boundary interface as seen in Fig. 1.

The theoretical results are plotted as solid symbols in Fig. 2b. Good qualitative support of the experimentally observed Mach number shift is indicated by the theoretical results computed at Mach = 1.26, 1.95, and 3.01 ($\gamma = 1.4$). Based upon this agreement the characteristics solutions were extended to include the effects of specific heat ratio γ on the freejet height. These results, also shown plotted in Fig. 2b, were calculated at Mach = 1.95 for $\gamma = 1.1$, 1.4, and 1.67 and clearly indicate the influence of γ.

Analysis of Results

It was found that all of the data, which is plotted on Fig. 2b, would collapse onto a single curve when expressed in terms of the dimensionless height, h/b_e, and the product of Mach number and pressure ratio, $M_e P_e/P_b$ as opposed to the dimensionless back pressure ratio $\gamma_e M_e^2 P_e/P_b$. This has been done on Fig. 3 where the axisymmetric freejet data correlation of Ref. 2 has also been included for completeness. Note that the abscissa in Fig. 3 has been generalized to accomodate both axisymmetric and two-dimensional data. The index number j is understood to be one or zero depending upon whether the jet flow is axisymmetric or two-dimensional, respectively. The final universal curve which results from this correlation of data can be expressed as

$$h/b_e = [1/(j+1)](\gamma_e M_e^2 P_e/P_b)^{1/(j+1)}(\gamma_e M_e)^{j-1} \qquad (1)$$

This new result implies that for the two-dimensional freejet case, the dimensionless jet height is independent of the specific heat ratio γ and is only linearly dependent upon the jet exit Mach number.

The observation that the two-dimensional jet height is independent of γ was previously pointed out by Vinson[6] as a result of his computer solutions. This matter was discussed in Ref. 2. Recently, this same observation has been indirectly verified by Thayer and Corlett[7] in their work with the two-dimensional jet interaction problem. They observed experimentally that the flowfield was "insensitive to changes in the injectant specific heat ratio."

The work of Werle et al.[1] indirectly supports the linear dependence of jet Mach number with jet height. The use of Eq. (1) in the two-dimensional jet interaction model of Ref. (1) correctly predicts a decrease of jet amplification with an increase of jet exit Mach number in contradistinction to the recognized inconsistency previously noted in Ref. (1).

Conclusion

The current results are expressed by Eq. (1). It indicates that the jet height varies linearly with jet Mach number for both the axisymmetric and the two-dimensional flow cases. Further, in Eq. (1) the injectant specific heat ratio varies to the one-half power in the axisymmetric case while in the two-dimensional case the jet height is independent of γ.

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Boundary-Layer Effects on Pressure Variations in Ludwieg Tubes

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Nomenclature

a = speed of sound
d = inner diameter of storage tube
L = length of storage tube
M = Mach number
p = (static) pressure
p_S = stagnation pressure
r = inner radius of storage tube
Re_d = Reynolds number (formed with d)
t = time
δ_N = boundary-layer thickness at nozzle inlet
ν_{00} = kinematic viscosity at room temperature and atmospheric pressure
Δ = signifies difference

Subscripts
0 = initial (storage) state
1 = state in ideal flow behind expansion fan
at = atmospheric

Introduction

LUDWIEG Tubes are rather simple gasdynamic testing facilities in which almost steady sub- and supersonic flow can be maintained for some period of time. Basic to the operation cycle of a Ludwieg Tube is an expansion wave which propagates down a cylindrical tube serving as gas storage. The difference between the real and the ideal behavior of the flow behind the expansion wave is caused by the finite viscosity and conductivity of the operating gas, usually air.

After the head of the expansion wave has passed some point of the storage tube a boundary layer develops at the tube wall by which mass is being transported from the near-wall region into the remaining potential core of the flow. When the boundary layer has grown together tube flow is assumed. In both cases waves are created travelling up- and downstream through the tube and changing the fluid-mechanical and thermodynamic state of the gas. Especially the flow ahead of the nozzle is accelerated by these waves. As the boundary condition $M = 1$ in the throat of the nozzle will be maintained in quasistationary flow part of the oncoming waves will be reflected. Only the net effect of these concurring wave systems can be measured at any particular point in the storage tube. The boundary-layer growth and the resulting variations of pressure and stagnation pressure were calculated in the theory[1] by E. Becker.

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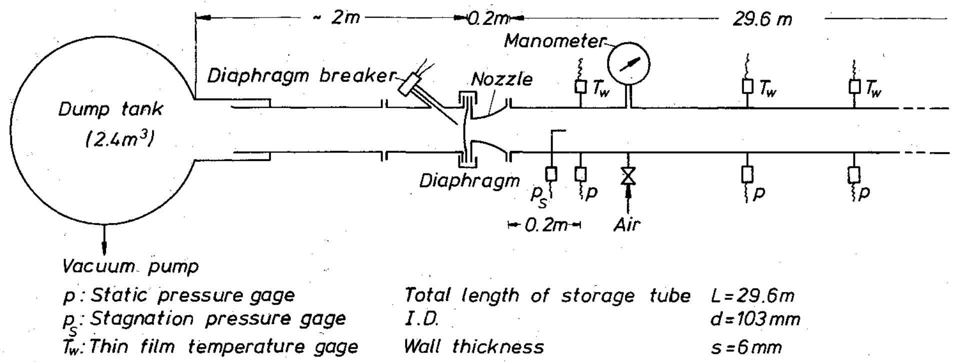

Fig. 1 Set-up of the Ludwieg Tube.

In the following, experimental results will be given to test this theory. It will be shown that it should be modified to give satisfactory agreement with experimental data. Basic assumptions being made are: 1) low Mach-numbers ($M_1 \leq 0.4$) in the storage tube; 2) essentially turbulent flow behind the expansion fan; this being a realistic assumption in high Reynolds number testing.

Description of Apparatus and Experiments

The experiments were performed in a cylindrical pilot tube with the set-up of Fig. 1. In order to obtain information for the cases of thin as well as thick boundary layers the tube was chosen unusually long ($L/d = 288$). By either evacuating the dump tank or/and increasing the initial storage pressure ($p_{omax} = 1.1 \cdot 10^6\ N/m^2$) and by using nozzles with different throat areas, experiments could be made within a Mach number range between $M_1 = 0.065$ and 0.405 while the Reynolds number varied from $Re_d = 2 \cdot 10^5$ to $6 \cdot 10^6$.

In particular the following quantities were measured: The pressure by means of piezoelectric gages at the tube wall at different distances from the nozzle, especially at the nozzle inlet where the stagnation pressure on the tube axis was registrated also. The experimental data that are obtained at the nozzle entrance are a good estimate for the corresponding quantities in the testing chamber which is usually connected to the downstream end of the nozzle. In addition, the history of the wall temperature at some places of the tube was measured by means of thin film elements. From these records one can determine the instant of time at which the initially laminar boundary layer becomes turbulent.

Experimental Results

Typical records are shown in Fig. 2; of interest for the purpose of this paper is the behavior of pressure p and stagnation pressure p_S ahead of the nozzle as function of time. The other curves, labeled T_w and $\dot{q}_w$, show the behavior of wall temperature and wall heat flux and will not be discussed in this paper. It was found that the pressure p always decreases linearly with time within the test period. The behavior of the stagnation pressure p_S on the tube axis at the nozzle entrance is more complicated. Shortly after opening of the diaphragm, the signal is relatively quiet; here the measurement is made in the potential core of the flow (clean tunnel). During this period which is characterized by relatively thin growing boundary layers and is thus of prime importance to the operation of Ludwieg Tubes, the stagnation pressure may decrease with time, as happens for low Mach numbers up to about $M_1 = 0.3$ or it may increase as happens for higher values of M_1. As time passes by, the signal gradually becomes noisy because turbulent fluctuations now reach the gage. Suddenly the mean signal turns down. It appears that the boundary layer has grown together at this instant although the velocity and temperature profiles seem to have not yet reached their final shapes. Usually the bend is followed by a period with a larger drop in stagnation pressure where the profiles adjust to their final forms. In the last part of the test period we have fully developed turbulent tube flow ahead of the nozzle. Now static and stagnation pressures decrease linearly with time and their relative changes are equal as is to be expected from theoretical considerations.

Comparison with Theory

In order to compare the experimental results with Becker's theory, the period with thin and growing boundary layer has to be treated differently than the period with fully developed tube flow. In the latter case there is a good agreement between theory and experiment. In the prior case the agreement is poorer. Becker's theory throughout predicts net variations of pressure and stagnation pressure that are considerably larger than the

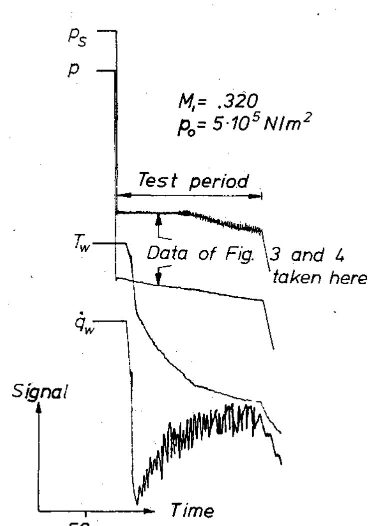

Fig. 2 Typical behavior of measured quantities.

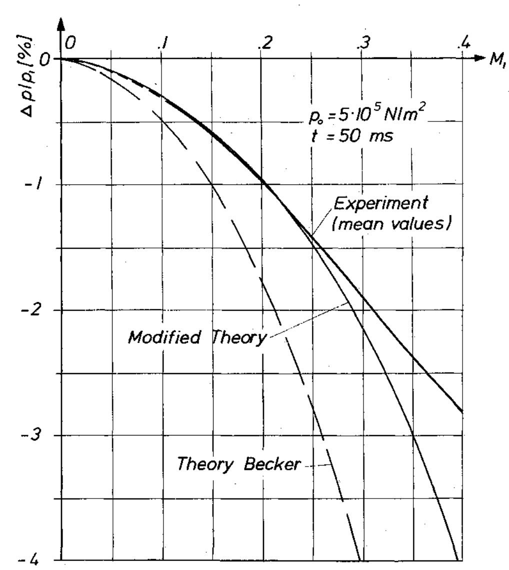

Fig. 3 Static pressure variation vs Mach number.

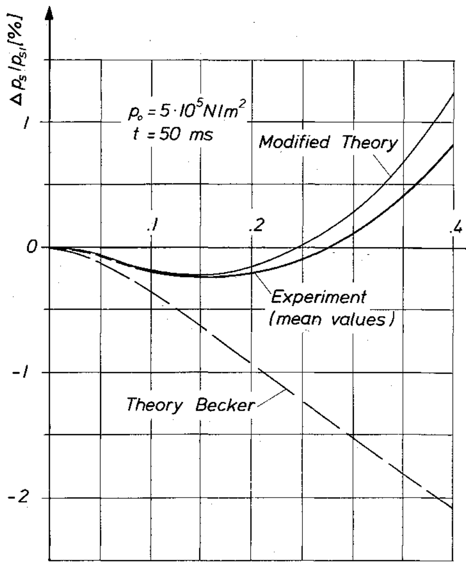

Fig. 4 Stagnation pressure variation vs Mach number.

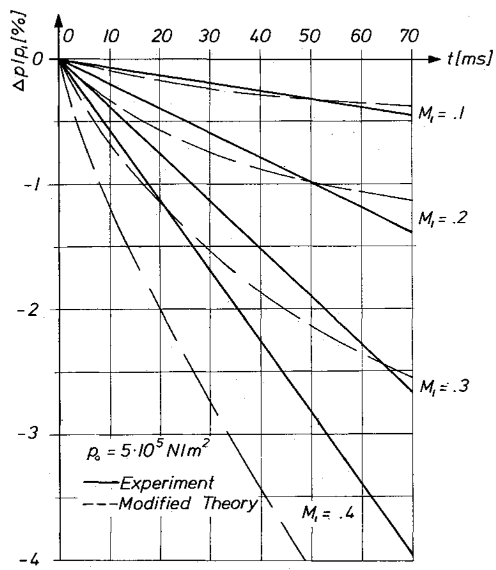

Fig. 5 Static pressure vs time.

measured. Therefore the original theory should be modified by accepting some different assumptions. One of the most important modifications was the inclusion of the curvature-effect of the tube wall on the mass sources which result from the boundary-layer growth. So the theoretical treatment can be extended to longer running times (thicker boundary layers). The results are typically shown in Fig. 3 and 4 for the variations of static and stagnation pressure ahead of the nozzle as functions of Mach number at a running time of 50 msec after opening of the diaphragm (see Fig. 2). The different curves shown follow mean values of the experimental data, Becker's original theory, and the modified theory which is presented in detail in Ref. 2. For low Mach numbers the agreement between the modified theory and the experiments is good. It should not be concealed, however, that the parameter t chosen in Fig. 3 and 4 leads to rather favorable results. For other running times the agreement is poorer. This can be seen from Fig. 5 for the static pressure variation at different Mach numbers. For short running times the pressure drop is overestimated. The reason is that the flow is assumed to be set into motion by an "expansion shock" of zero extent rather than a finite width expansion wave. Later this effect is lessened to give a favorable prediction of the pressure variation.

Summary

To estimate the variations of static and stagnation pressure at the nozzle inlet of a Ludwieg Tube due to boundary-layer growth Becker's modified theory is recommended as it is described in Ref. 2. Moreover the variations of other thermodynamic properties of the operating gas can be computed without difficulty. By systematic linearization in terms of the Mach number M_1 the numerical treatment of Becker's theory is considerably simplified and can be handled on a desk computer.

For calculation which is valid for a specific heat ratio $\gamma = 1.4$ the following steps have to be taken. 1) Boundary-layer growth at the nozzle (Kàrmàn-Pohlhausen-Method)

$$\delta_N(t) = 0.303 M_1^{3/5}(1 - 0.623 M_1)\{(\nu_{oo}/a_o)(p_{at}/p_o)\}^{1/5}(a_o t)^{4/5}$$

2) From the solution of the inhomogeneous wave equation

$$\Delta p'/p_1 = 0.123 M_1(1 - 0.064 M_1)(1 + \tfrac{1}{2}\delta_N/r + \tfrac{1}{3}\delta_N^2/r^2)\,\delta_N/r$$

(primary compression waves being directly caused by the boundary-layer). 3) Boundary condition $M = 1$ in the nozzle throat

$$\frac{M(t)}{[1 + 0.2M(t)^2]^3} = \frac{M_1}{(1 + 0.2M_1^2)^3}\left(\frac{1 - \varphi(M_1)\,\delta_N^*/r}{1 - \delta_N^*/r}\right)^2$$

where

$$\varphi(M_1) = 1.31 M_1^{5/2}(1 - 0.24 M_1); \quad \delta_N^* = 0.125(1 + 0.311 M_1)\delta_N$$

4) Because of reflected waves at the nozzle

$$\frac{\Delta p''}{p_1} = \frac{7}{5}\,\frac{M_1 + \frac{5}{7}\Delta p'/p_1 - M(t)(1 - \frac{1}{7}\Delta p'/p_1)}{1 + 0.2M(t)}$$

5) Resultant pressure and stagnation pressure variations are

$$\Delta p/p_1 = (\Delta p' + \Delta p'')/p_1$$

and

$$\Delta p_S/p_{S1} = \left[\frac{1 + 0.2M(t)^2}{1 + 0.2M_1^2}\right]^{3.5}(1 + \Delta p/p_1) - 1$$

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Proper Equations and Similar Approximations in the Hypersonic Merged Layer

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Nomenclature

f = nondimensional stream function
H = total enthalpy
M_∞ = freestream Mach number
p = static pressure
Pr = Prandtl number
Re/x = Reynolds number per unit length x
U_∞ = freestream velocity
T = static temperature
u = streamwise velocity component
v = normal velocity component
x = streamwise coordinate

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